On redefinitions of variables in gauge field theory

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Abstract

In this paper, for massive fields of spins 2 and 3 with non-canonical Lagrangians, we build Hamiltonians and full systems of constraints and show that the use of derivatives in a redefinition of fields can give rise to a change of number of physical degrees of freedom.

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Introduction

At constructing various kind of field theory models it is often useful to redefine initial fields for the theory to be of simpler and more understandable form. Such substitutions of variables must not change physical contents of the model i.e. the number of physical degrees of freedom must remain the same as before the substitution. In this, of course, it is meant that a modification of Poincare group representations didn't happen i.e. for example, a massive vector field does not turn into three scalar fields. One has often to perform such type of redefinitions in theories, which describe a physical particle with some set of fields, see Ref. [1, 2]. So, for instance, in [2] when describing massive spin-2 particle propagation in a homogeneous electromagnetic field the result independent of space-time dimensionality has been obtained using a redefinition of second rank field.

One can divide all substitutions of variables into two kinds. First kind are the substitutions of variables without derivatives i.e. schematically $\Phi'_A = M_A^B \Phi_B + F_A^{BC} \Phi_B \Phi_C + \ldots$, where M_A^B is non-degenerate matrix. The second kind are substitutions with derivatives i.e. they have form $\Phi'_A = M_A^B \Phi_B + H_A^B \partial \Phi_B + \ldots$. In this paper using the case of free massive spin-3 field, we show that the number of physical degrees of freedom of the theory can change, if one uses derivatives in the redefinition of fields.

To begin with in Section 1 we consider the free massive spin-2 field that is described with a non-canonical Lagrangian[1] derived from the canonical form with the redefinition of the second rank field. We build a canonical Hamiltonian and a full system of constraints[2] (all the constraints are of the first kind). A simple calculation shows that the number of degrees of freedom remains the same at transition to the non-canonical form.

In Section 2 we are building a full system of constraints and canonical Hamiltonian for a non-canonical Lagrangian, which describes free massive spin-3 field and which is derived from the canonical form with the redefinition of fields without derivatives. In this case all the constraints are of the first kind and the number of degrees of freedom remains the same as in the canonical case.

In Section 3 we consider the field of spin 3 in the non-canonical form that has been obtained from the canonical one with a redefinition of fields with the use of the derivative. Building a Hamiltonian and full systems of constraints, we show that in this case the number of field degrees of freedom increases. In this the constraints of the second kind are present in a full system.

1 Free Massive Field with spin 2

At first, let us consider the spin-2 field to compare with the case of spin-3 field.

We consider the usual flat Minkowski space $\mathbf{M}^4$ with metric signature $(1, -1, -1, -1)$. Latin indices take the value $k, l, \ldots = 0, 1, 2, 3$ and the Greek ones — the value $\alpha, \beta, \ldots = 1, 2, 3$. For convenience we will not make difference between upper and lower indices, while

[1] We call a Lagrangian of free massive spin-s field as canonical, if it breaks into the sum of Lagrangians for massless fields of spins $s, s-1, ..., 0$ in the massless limit, see [2].

[2] Describing systems with constraints, we use standard Dirac procedure, ref. [3, 4].

the summation over the repeated indices will be understood, as usual, i.e.

$$A_{k\ldots}B_{k\ldots} \equiv g^{kl}A_{k\ldots}B_{l\ldots}.$$

We will describe the free massive field of spin 2 with the gauge invariant Lagrangian of type

$$\begin{aligned} \mathcal{L}_0 &= \partial_m \bar{h}_{kl}\partial_m h_{kl} - 2\partial_k h_{kl}\partial_m \bar{h}_{lm} + \left(\partial_k h_{kl}\partial_l \bar{h} + h.c.\right) - \partial_k \bar{h}\partial_k h \\ &+ 2\left(\partial_k \bar{h}_{kl}\partial_l \varphi - \partial_l \bar{h}\partial_l \varphi + h.c.\right) - 2\left(\partial_l \bar{b}_k \partial_l b_k - \partial_l b_k \partial_k \bar{b}_l\right) \\ &+ 2m\left(\partial_l \bar{b}_k h_{kl} - \partial_k \bar{b}_k h + h.c.\right) - m^2\left(\bar{h}_{kl}h_{kl} - \bar{h}h\right) , \end{aligned} \tag{1}$$

where h_{kl} is a symmetrical tensor and $h = g^{kl}h_{kl}$.

The gauge transformations of the fields have the following form:

$$\begin{aligned} \delta h_{kl} &= 2\partial_{(k}\xi_{l)}, \\ \delta b_k &= \partial_k \eta + m\xi_k, \\ \delta\varphi &= m\eta . \end{aligned} \tag{2}$$

Lagrangian (1) has been chosen in a non-canonical form (with the off-diagonal kinetic part) in order that the Goldstone part (proportional to mass) for the field h_{kl} be absent in the transformations. The transformations for h has the form $\delta h_{kl} = 2\partial_{(k}\xi_{l)} + mg_{kl}\eta$, where g_{kl} is the metrical tensor, in the canonical form with the same normalization of fields. Therefore, to pass to Lagrangian (1) and transformations (2) one need do the following substitution of variables $h'_{kl} \rightarrow h_{kl} - g_{kl}\varphi$.

Further on, for convenience, we put $m = 1$.

Passing from Lagrangian (1) to the Hamiltonian formalism, we get the following five constraints calculating the momenta

$$\begin{aligned} \overset{(1)}{C}{}^h_\alpha &= p^h_{\alpha 0} - \partial_\alpha h_{\beta\beta} + 2\partial_\beta h_{\alpha\beta} + 2\partial_\alpha \varphi + \partial_\alpha h_{00}, \\ \overset{(1)}{C}{}^h &= p^h_{00} - \partial_\alpha h_{\alpha 0}, \\ \overset{(1)}{C}{}^b &= p^b_0 - 2h_{\alpha\alpha}. \end{aligned} \tag{3}$$

Let us define the Poisson brackets in the following form:

$$\begin{aligned} \{h^{\alpha\beta}(x), p^h_{\mu\nu}(y)\} &= \delta^{\alpha\beta}_{(\mu\nu)}(x-y), \\ \{h^{\alpha 0}(x), p^h_{\beta 0}(y)\} &= \delta^\alpha_\beta(x-y), \\ \{h^{00}(x), p^h_{00}(y)\} &= \delta(x-y), \\ \{b^\alpha(x), p^b_\beta(y)\} &= \delta^\alpha_\beta(x-y), \\ \{b^0(x), p^b_0(y)\} &= \delta(x-y), \\ \{\varphi(x), p^\varphi(y)\} &= \delta(x-y) , \end{aligned} \tag{4}$$

where we use the notation $\delta^{\alpha\beta\ldots}_{\mu\nu\ldots}(x-y) \equiv \delta^\alpha_\mu \delta^{\beta\ldots}_{\nu\ldots}\delta(x-y)$.

The Poisson brackets of constraints (3) equal zero between themselves. At that the Hamiltonian obtained from (1) has the following form:

$$
\begin{aligned}
\mathcal{H}_0 \ = \ & \bar{p}^h_{\alpha\beta}p^h_{\alpha\beta} - \frac{1}{3}\bar{p}^h_{\beta\beta}p^h_{\alpha\alpha} + \frac{1}{6}\bar{p}^h_{\alpha\alpha}p^\varphi + \frac{1}{6}p^h_{\alpha\alpha}\bar{p}^\varphi + \frac{1}{2}\bar{p}^b_\alpha p^b_\alpha + \frac{1}{6}\bar{p}^\varphi p^\varphi \\
& + \frac{1}{3}\partial_\alpha h_{\alpha 0}\bar{p}^h_{\beta\beta} + \frac{1}{3}\partial_\alpha \bar{h}_{\alpha 0}p^h_{\beta\beta} + \partial_\alpha \bar{b}_0 p^b_\alpha + \partial_\alpha b_0 \bar{p}^b_\alpha - \frac{1}{6}\partial_\alpha h_{\alpha 0}\bar{p}^\varphi \\
& - \frac{1}{6}\partial_\alpha \bar{h}_{\alpha 0}p^\varphi + \bar{h}_{\alpha 0}p^b_\alpha + h_{\alpha 0}\bar{p}^b_\alpha - \partial_\alpha h_{00}\partial_\beta \bar{h}_{\alpha\beta} - \partial_\beta \bar{h}_{00}\partial_\alpha h_{\alpha\beta} \\
& + \partial_\beta h_{\alpha\alpha}\partial_\beta \bar{h}_{00} + \partial_\alpha h_{00}\partial_\alpha \bar{h}_{\beta\beta} + \frac{2}{3}\partial_\alpha h_{\alpha 0}\partial_\beta \bar{h}_{\beta 0} - 2\partial_\beta h_{\alpha 0}\partial_\beta \bar{h}_{\alpha 0} \\
& + \partial_\gamma h_{\alpha\beta}\partial_\gamma \bar{h}_{\alpha\beta} + \partial_\beta h_{\alpha\alpha}\partial_\gamma \bar{h}_{\beta\gamma} + \partial_\beta \bar{h}_{\gamma\gamma}\partial_\alpha h_{\alpha\beta} - \partial_\beta h_{\alpha\alpha}\partial_\beta \bar{h}_{\gamma\gamma} \\
& - 2\partial_\gamma \bar{h}_{\beta\gamma}\partial_\alpha h_{\alpha\beta} - 2\partial_\alpha \varphi \partial_\alpha \bar{h}_{00} - 2\partial_\alpha \bar{\varphi}\partial_\alpha h_{00} + 2\partial_\beta h_{\alpha\alpha}\partial_\beta \bar{\varphi} \\
& + 2\partial_\beta \bar{h}_{\alpha\alpha}\partial_\beta \varphi - 2\partial_\beta \bar{\varphi}\partial_\alpha h_{\alpha\beta} - 2\partial_\alpha \bar{h}_{\alpha\beta}\partial_\beta \varphi + 2\partial_\beta b_\alpha \partial_\beta \bar{b}_\alpha \\
& - 2\partial_\beta b_\alpha \partial_\alpha \bar{b}_\beta - 2\partial_\alpha b_\alpha \bar{h}_{00} - 2\partial_\alpha \bar{b}_\alpha h_{00} - 2\partial_\beta b_\alpha \bar{h}_{\alpha\beta} - 2\partial_\beta \bar{b}_\alpha h_{\alpha\beta} \\
& + 2\partial_\alpha b_\alpha \bar{h}_{\beta\beta} + 2\partial_\alpha \bar{b}_\alpha h_{\beta\beta} + 4\partial_\alpha \bar{b}_0 h_{\alpha 0} + 4\partial_\alpha b_0 \bar{h}_{\alpha 0} + \bar{h}_{\alpha\beta}h_{\alpha\beta} \\
& + \bar{h}_{00}h_{\alpha\alpha} + \bar{h}_{\alpha\alpha}h_{00} - \bar{h}_{\beta\beta}h_{\alpha\alpha} \ ,
\end{aligned}
\tag{5}
$$

where $\gamma_{\alpha\beta} = -g_{\alpha\beta}$ and $A_{\alpha\alpha\ldots} \equiv \gamma^{\alpha\beta}A_{\alpha\beta\ldots}$.

In order that the Hamiltonian equations be equivalent to the Lagrangian ones followed from (1), one has to add the first step constraints to the Hamiltonian, but since the constraints commute between themselves one needn't add it to Hamiltonian for the calculation of second step constraints.

At the second stage we get 5 second step constraints, calculating the evolution of first step ones

$$
\begin{aligned}
\overset{(2)}{C}{}^h_\alpha \ &= \ 2\partial_\beta p^h_{\alpha\beta} \ - \ p^b_\alpha \ - \ 2\Delta h_{\alpha 0} \ - \ 4\partial_\alpha b_0 \\
\overset{(2)}{C}{}^h \ &= \ - \ \Delta h_{\alpha\alpha} \ + \ \partial_{\alpha\beta}h_{\alpha\beta} \ + \ 2\Delta\varphi \ - \ 2\partial_\alpha b_\alpha \ + \ h_{\alpha\alpha} \\
\overset{(2)}{C}{}^b \ &= \ \partial_\alpha p^b_\alpha \ - \ p^\varphi \ + \ 2\partial_\alpha h_{\alpha 0} \ .
\end{aligned}
\tag{6}
$$

The Poisson brackets equal zero among the second step constraints and between them and the first step ones.

The brackets between constraints (6) and Hamiltonian (5) equal either zero or linear combinations of second step constraints. That is, new constraints do not appear at the third stage. Hence (3) and (6) form the full system of constraints for this theory. In this, all the constraints are the first kind ones.

It is easy to compute that the number of degrees of freedom equal five in this case. This agrees with the formula $2s + 1$ for a massive particle of arbitrary spin s.

2 Free massive field of spin 3: substitution of variables without derivatives

As in the previous Section we will describe a free massive complex field of spin 3 with the gauge invariant Lagrangian in the non-canonical form

$$
\begin{aligned}
\mathcal{L}_0 \ = \ & -10\partial_n\bar{\Phi}_{klm}\partial_n\Phi_{klm} + 30\partial_k\Phi_{klm}\partial_n\bar{\Phi}_{lmn} - 30\left(\partial_k\Phi_{klm}\partial_m\bar{\Phi}_l + h.c.\right) \\
& + 30\partial_l\Phi_k\partial_l\bar{\Phi}_k + 15\partial_l\bar{\Phi}_l\partial_k\Phi_k - 6\Big(2\partial_k\Phi_{klm}\partial_m\bar{b}_l - 2\partial_l\Phi_k\partial_l\bar{b}_k \\
& - \partial_l\bar{\Phi}_l\partial_k b_k + h.c.\Big) + \frac{36}{5}\partial_l b_k\partial_k\bar{b}_l + 30\partial_m\bar{h}_{kl}\partial_m h_{kl} - 60\partial_k h_{kl}\partial_m\bar{h}_{lm} \\
& + 30\left(\partial_l\bar{h}\partial_k h_{kl} + h.c.\right) - 30\partial_k h\partial_k\bar{h} + 5\left(\partial_l\bar{\varphi}\partial_k h_{kl} - \partial_k h\partial_k\bar{\varphi} + h.c.\right) \\
& - \frac{1}{4}\partial_k\bar{\varphi}\partial_k\varphi - 15\left(2\partial_m\bar{h}_{kl}\Phi_{klm} - 4\partial_k\bar{h}_{kl}\Phi_l + \partial_k\bar{h}\Phi_k + h.c.\right) \\
& - \frac{5}{2}\left(\partial_k\bar{\varphi}\Phi_k + h.c.\right) - 18\left(\partial_k\bar{b}_k h + h.c.\right) + 5\Big(2\bar{\Phi}_{klm}\Phi_{klm} \\
& - 6\bar{\Phi}_k\Phi_k + 9\bar{h}h\Big) \ , \qquad (7)
\end{aligned}
$$

where Φ_{klm} is the symmetric tensor and $\Phi_k \overset{def}{=} g^{lm}\Phi_{klm}$.

The transformations of the fields for this Lagrangian have the following form:

$$
\begin{aligned}
\delta\Phi_{klm} &= 3\partial_{(k}\omega_{lm)} - \tfrac{3}{5}g_{(kl}\partial_{m)}\eta, \\
\delta h_{kl} &= 2\partial_{(k}\xi_{l)} + \omega_{kl}, \\
\delta b_k &= 2\partial_k\eta + 5\xi_k, \\
\delta\varphi &= 12\eta,
\end{aligned} \qquad (8)
$$

at that $g^{kl}\omega_{kl} = 0$

The transformations of rank 2 and 3 fields have the form of type $\delta\Phi = \partial\omega + g\,\xi$ and $\delta h = \partial\xi + \omega + g\,\eta$. It is evident that transformations (8) looks simpler, moreover the Goldstone part for the field Φ_{klm} is absent in the transformations. This facilitate an analysis of the theory at switching on interaction. The transition from the canonical form to Lagrangian (7) and transformations (8) has been reached with the fields redefinitions of type

$$
\begin{aligned}
\Phi' &\rightarrow \Phi - g\,b, \\
h' &\rightarrow h - g\,\varphi\ .
\end{aligned} \qquad (9)
$$

In order to show that the number of degrees of freedom remains the same we will compute the constraint algebra of theory (7).

Calculating the canonically conjugated momenta we obtain 14 first step constraints

$$
\begin{aligned}
\overset{(1)}{C}{}^{b} &= \frac{6}{5}p^{\Phi}_{000} - p^{b}_{0} - 12\partial_\delta\Phi_{\gamma\gamma\delta} + 24\partial_\gamma\Phi_{\gamma 00} + \frac{36}{5}\partial_\gamma b_\gamma, \\
\overset{(1)}{C}{}^{h} &= -p^{h}_{00} - 45\Phi_{\gamma\gamma 0} + 15\Phi_{000} + 30\partial_\gamma h_{\gamma 0} + 18 b_0, \qquad (10) \\
\overset{(1)}{C}{}^{h}_{\alpha} &= -p^{h}_{\alpha 0} + 60\Phi_{\alpha\gamma\gamma} - 60\partial_\gamma h_{\alpha\gamma} + 30\partial_\alpha h_{\gamma\gamma} - 30\partial_\alpha h_{00} - 5\partial_\alpha\varphi,
\end{aligned}
$$

$$
\begin{aligned}
\overset{(1)}{C}{}^{\Phi}_{\alpha} &= -p^{\Phi}_{\alpha 00} + 30\partial_\gamma \Phi_{\alpha\gamma 0} - 30\partial_\alpha \Phi_{\gamma\gamma 0} + 30\partial_\alpha \Phi_{000} + 12\partial_\alpha b_0, \\
\overset{(1)}{C}{}^{\Phi}_{\alpha\beta} &= -p^{\Phi}_{\alpha\beta 0} - 3\gamma_{\alpha\beta} p^{\Phi}_{000} - 30\partial_\gamma \Phi_{\alpha\beta\gamma} + 30\partial_{(\alpha}\Phi_{\beta)\gamma\gamma} - 30\partial_{(\alpha}\Phi_{\beta)00} \\
&\quad + 30\gamma_{\alpha\beta}\partial_\delta \Phi_{\gamma\gamma\delta} - 60\gamma_{\alpha\beta}\partial_\gamma \Phi_{\gamma 00} - 12\partial_{(\alpha} b_{\beta)} - 12\gamma_{\alpha\beta}\partial_\gamma b_\gamma \ .
\end{aligned}
$$

Let us update Poisson brackets (4)

$$
\begin{aligned}
\{\Phi^{\alpha\beta\gamma}(x), p^{\Phi}_{\lambda\mu\nu}(y)\} &= \delta^{\alpha\beta\gamma}_{(\lambda\mu\nu)}(x-y), \\
\{\Phi^{\alpha\beta 0}(x), p^{\Phi}_{\mu\nu 0}(y)\} &= \delta^{\alpha\beta}_{(\mu\nu)}(x-y), \\
\{\Phi^{\alpha 00}(x), p^{\Phi}_{\beta 00}(y)\} &= \delta^{\alpha}_{\beta}(x-y), \\
\{\Phi^{000}(x), p^{\Phi}_{000}(y)\} &= \delta(x-y) \ .
\end{aligned} \tag{11}
$$

The Poisson brackets of all the first step constraints equal zero among themselves.

Now we need to compute the canonical Hamiltonian. The result is rather cumbersome even for free field, therefore, we place the concrete expression for the Hamiltonian in Appendix A.

From the condition of conservation of the first step constraints, we get the second step constraints

$$
\begin{aligned}
\overset{(2)}{C}{}^{b} &= -\partial_\gamma p^{b}_{\gamma} - 0.1 p^{h}_{\gamma\gamma} - 5.4 b_0 + 6p^{\varphi} - 24\partial^2 \Phi_{\gamma\gamma 0} + 12\partial^2_{\gamma\delta}\Phi_{\gamma\delta 0} \\
&\quad + 12\partial^2 \Phi_{000} + 7.2\partial^2 b_0 - 33\partial_\gamma h_{\gamma 0} + 13.5\Phi_{\gamma\gamma 0} - 4.5\Phi_{000}, \\
\overset{(2)}{C}{}^{h} &= 3p^{\Phi}_{000} - 30\partial^2 h_{\gamma\gamma} + 30\partial^2_{\gamma\delta} h_{\gamma\delta} + 5\partial^2\varphi - 60\partial_\delta \Phi_{\gamma\gamma\delta} + 90\partial_\gamma \Phi_{\gamma 00} \\
&\quad + 45 h_{\gamma\gamma} - 45 h_{00}, \\
\overset{(2)}{C}{}^{h}_{\alpha} &= -2\partial_\gamma p^{h}_{\alpha\gamma} + 5p^{b}_{\alpha} + 60\partial^2 h_{\alpha 0} - 60\partial_\gamma \Phi_{\alpha\gamma 0} + 30\partial_\alpha \Phi_{\gamma\gamma 0} \\
&\quad - 30\partial_\alpha \Phi_{000}, \\
\overset{(2)}{C}{}^{\Phi}_{\alpha} &= 3\partial_\alpha p^{\Phi}_{000} - 30\partial^2 \Phi_{\alpha\gamma\gamma} + 30\partial^2_{\gamma\delta}\Phi_{\alpha\gamma\delta} - 60\partial^2_{\alpha\delta}\Phi_{\gamma\gamma\delta} + 90\partial^2_{\alpha\gamma}\Phi_{\gamma 00} \\
&\quad + 12\partial^2 b_\alpha + 24\partial^2_{\alpha\gamma} b_\gamma + 30\Phi_{\alpha\gamma\gamma} - 60\partial_\gamma h_{\alpha\gamma} + 15\partial_\alpha h_{\gamma\gamma} \\
&\quad - 45\partial_\alpha h_{00} - \frac{5}{2}\partial_\alpha \varphi, \\
\overset{(2)}{C}{}^{\Phi}_{\alpha\beta} &= -3\partial_\gamma p^{\Phi}_{\alpha\beta\gamma} + p^{h}_{\alpha\beta} + 30\partial^2 \Phi_{\alpha\beta 0} - 30\partial^2_{\alpha\beta}\Phi_{000} + 30\partial^2_{\alpha\beta}\Phi_{\gamma\gamma 0} \\
&\quad - 30\gamma_{\alpha\beta}\partial^2_{\gamma\delta}\Phi_{\gamma\delta 0} - 30\gamma_{\alpha\beta}\partial^2\Phi_{000} + 60\gamma_{\alpha\beta}\partial^2\Phi_{\gamma\gamma 0} - 12\partial^2_{\alpha\beta} b_0 \\
&\quad - 24\gamma_{\alpha\beta}\partial^2 b_0 + 60\partial_{(\alpha} h_{\beta)0} + 90\gamma_{\alpha\beta}\partial_\gamma h_{\gamma 0} - 45\gamma_{\alpha\beta}\Phi_{\gamma\gamma 0} \\
&\quad + 15\gamma_{\alpha\beta}\Phi_{000} + 18\gamma_{\alpha\beta} b_0 \ .
\end{aligned} \tag{12}
$$

The second step constraints have zero brackets among themselves and between them and the first step ones. New constraints do not appear at the third stage. Hence (10) and (12) constitute the full system of constraints. In this, all the constraints are of the first kind.

It is easy to compute the number of independent field degrees of freedom. The number of all field components equals 35 and number of the constraints 28, therefore, the number of independent degrees of freedom equals $35 - 28 = 7$. Thus passing to the non-canonical form (7) with the substitution of variables (9), the number of degrees of freedom has not changed.

3 Massive spin-3 field: substitution with derivatives

When looking at transformations (8) a desire arises to simplify the ones making a third rank field shift of type

$$\Phi' \to \Phi + g\partial\varphi \tag{13}$$

so that the transformations for Φ remain only of type $\partial\omega$. Besides, simplicity of the transformations gives us another advantage. Since the metrical tensor is absent in the transformations after such shift, the Lagrangian does not depend on the space-time dimensionality.

However the Lagrangian becomes the third degree one in derivatives. The question emerges whether the number of physical degrees of freedom changes at that.

Let us show that the number of degrees of freedom increases by one at the redefinitions of type (13).

In order to reduce the number of derivatives in the Lagrangian we introduce an auxiliary field v_k. In this, the Lagrangian acquire the following form

$$\begin{aligned} \mathcal{L}_0 \;=\; & (2\partial_m\Phi_{klm}\partial_l\bar{v}_k - 2\partial_l\Phi_k\partial_l\bar{v}_k + 2\partial_l\Phi_k\partial_k\bar{v}_l - 3\partial_l\Phi_l\partial_m\bar{v}_m + h.c.) \\ & - 10\partial_n\Phi_{klm}\partial_n\bar{\Phi}_{klm} + 30\partial_n\Phi_{kln}\partial_m\bar{\Phi}_{klm} - 30\left(\partial_n\Phi_{kmn}\partial_m\bar{\Phi}_k + h.c.\right) \\ & + 30\partial_m\Phi_k\partial_m\bar{\Phi}_k + 15\partial_m\Phi_m\partial_k\bar{\Phi}_k - 6\left(2\partial_m\Phi_{klm}\partial_l\bar{b}_k - 2\partial_m b_l\partial_m\bar{\Phi}_l\right. \\ & \left. - \partial_m b_m\partial_l\bar{\Phi}_l + h.c.\right) + 30\partial_m h_{kl}\partial_m\bar{h}_{kl} - 60\partial_m h_{km}\partial_l\bar{h}_{kl} \\ & + 30\left(\partial_m h_{lm}\partial_l\bar{h} + h.c.\right) - 30\partial_l h\partial_l\bar{h} - 15\left(2\partial_m h_{kl}\bar{\Phi}_{klm}\right. \\ & \left. - 4\partial_m h_{km}\bar{\Phi}_k + \partial_k h\bar{\Phi}_k + h.c.\right) + \left(\bar{\lambda}_k\left(\partial_k\varphi - v_k\right) + h.c.\right) \\ & + 10\Phi_{klm}\bar{\Phi}_{klm} - 30\Phi_k\bar{\Phi}_k\ . \end{aligned} \tag{14}$$

Correspondingly, gauge transformations (8) after shift (13) and entering the auxiliary field have the following form:

$$\begin{aligned} \delta\Phi_{klm} &= 3\partial_{(k}\omega_{lm)}, \\ \delta h_{kl} &= 2\partial_{(k}\xi_{l)} + \omega_{kl}, \\ \delta b_k &= 2\partial_k\eta + 5\xi_k, \\ \delta v_k &= 12\partial_k\eta, \\ \delta\varphi &= 12\eta\ . \end{aligned} \tag{15}$$

Passing to the Hamiltonian form of theory (14), we obtain the following constraints at this stage

$$
\begin{aligned}
\overset{(1)}{C}{}^{\lambda} &= p_0^{\lambda}, \\
\overset{(1)}{C}{}^{\lambda}_{\alpha} &= p_{\alpha}^{\lambda}, \\
\overset{(1)}{C}{}^{\varphi} &= p^{\varphi} - \lambda_0 \,, \\
\overset{(1)}{C}{}^{v} &= -\frac{1}{6}\, p_0^b - p_0^v - 2\partial_\delta \Phi_{\gamma\gamma\delta} + 2\partial_\gamma \Phi_{\gamma 00}, \\
\overset{(1)}{C}{}^{v}_{\alpha} &= -\frac{1}{6}\, p_\alpha^b - p_\alpha^v + 2\partial_\alpha \Phi_{\gamma\gamma 0} - 2\partial_\alpha \Phi_{000}, \\
\overset{(1)}{C}{}^{h} &= - p_{00}^h - 45\Phi_{\gamma\gamma 0} + 15\Phi_{000} + 30\partial_\gamma h_{\gamma 0}, \\
\overset{(1)}{C}{}^{h}_{\alpha} &= - p_{\alpha 0}^h + 60\Phi_{\alpha\gamma\gamma} - 60\partial_\gamma h_{\alpha\gamma} + 30\partial_\alpha h_{\gamma\gamma} - 30\partial_\alpha h_{00}, \\
\overset{(1)}{C}{}^{\Phi}_{\alpha} &= - p_{\alpha 00}^{\Phi} + 30\partial_\gamma \Phi_{\alpha\gamma 0} - 30\partial_\alpha \Phi_{\gamma\gamma 0}, \\
\overset{(1)}{C}{}^{\Phi}_{\alpha\beta} &= - p_{\alpha\beta 0}^{\Phi} - 3p_{000}^{\Phi}\gamma_{\alpha\beta} - 30\partial_\gamma \Phi_{\alpha\beta\gamma} - 30\partial_{(\alpha}\Phi_{\beta)00} + 30\partial_{(\alpha}\Phi_{\beta)\gamma\gamma} \\
&\quad + 30\partial_\delta \Phi_{\gamma\gamma\delta}\gamma_{\alpha\beta} - 60\partial_\gamma \Phi_{\gamma 00}\gamma_{\alpha\beta} - 12\partial_{(\alpha} b_{\beta)} - 12\partial_\gamma b_\gamma \gamma_{\alpha\beta} \\
&\quad + 2\partial_{(\alpha} v_{\beta)} + 6\partial_\gamma v_\gamma \gamma_{\alpha\beta} \,.
\end{aligned} \tag{16}
$$

Since unlike (7) the additional variables, namely, the auxiliary field v_k and the Lagrange multiplier λ_k arise in Lagrangian (14), therefore, one has to update the Poisson brackets

$$
\begin{aligned}
\{v_0\,(x)\,, p_0^v\,(y)\} &= \delta\,(x-y)\,, \\
\{v_\alpha\,(x)\,, p_\beta^v\,(y)\} &= \delta_{\alpha\beta}\,(x-y)\,, \\
\{\lambda_0\,(x)\,, p_0^\lambda\,(y)\} &= \delta\,(x-y)\,, \\
\{\lambda_\alpha\,(x)\,, p_\beta^\lambda\,(y)\} &= \delta_{\alpha\beta}\,(x-y)\ .
\end{aligned} \tag{17}
$$

There are only two the non-trivial brackets among the first step constraints

$$
\{\overset{(1)}{C}{}^{\lambda}\,(x)\,, \overset{(1)}{C}{}^{v}\,(y)\} = \delta\,(x-y)\ , \tag{18}
$$

hence, besides first kind constraints, the second kind ones emerge in the theory.

Canonical Hamiltonian obtained in this case is placed in appendix B.

From the condition of the first step constraint conservation, we obtain the second step constraints

$$
\begin{aligned}
\overset{(1)}{\Lambda}{}^{\lambda}_{0} &= \partial_\alpha \lambda_\alpha, \\
\overset{(1)}{\Lambda}{}^{\varphi} &= v_0, \\
\overset{(2)}{C}{}^{\lambda}_{\alpha} &= -\partial_\alpha \varphi + v_\alpha, \\
\overset{(2)}{C}{}^{v} &= \lambda_0,
\end{aligned}
$$

$$
\begin{aligned}
\overset{(2)}{C}{}^{v}_{\alpha} &= \lambda_\alpha, \\
\overset{(2)}{C}{}^{h} &= \frac{5}{2}p^b_0 - 30\partial^2 h_{\gamma\gamma} + 30\partial^2_{\gamma\delta}h_{\gamma\delta} - 30\partial_\delta\Phi_{\gamma\gamma\delta} + 30\partial_\gamma\Phi_{\gamma 00}, \\
\overset{(2)}{C}{}^{h}_{\alpha} &= -2\partial_\gamma p^h_{\alpha\gamma} + 5p^b_\alpha + 60\partial^2 h_{\alpha 0} - 60\partial_\gamma\Phi_{\alpha\gamma 0} + 30\partial_\alpha\Phi_{\gamma\gamma 0} \\
&\quad - 30\partial_\alpha\Phi_{000}, \\
\overset{(2)}{C}{}^{\Phi}_{\alpha} &= +3\partial_\alpha p^\Phi_{000} - 30\partial^2\Phi_{\alpha\gamma\gamma} + 30\partial^2_{\gamma\delta}\Phi_{\alpha\gamma\delta} - 60\partial^2_{\alpha\delta}\Phi_{\gamma\gamma\delta} + 90\partial^2_{\alpha\gamma}\Phi_{\gamma 00} \\
&\quad + 12\partial^2 b_\alpha + 24\partial^2_{\alpha\gamma}b_\gamma - 2\partial^2 v_\alpha - 10\partial^2_{\alpha\gamma}v_\gamma + 30\Phi_{\alpha\gamma\gamma} \\
&\quad - 60\partial_\gamma h_{\alpha\gamma} + 15\partial_\alpha h_{\gamma\gamma} - 45\partial_\alpha h_{00}, \\
\overset{(2)}{C}{}^{\Phi}_{\alpha\beta} &= -3\partial_\gamma p^\Phi_{\alpha\beta\gamma} + p^h_{\alpha\beta} + 30\partial^2\Phi_{\alpha\beta 0} - 30\partial^2_{\alpha\beta}\Phi_{000} + 30\partial^2_{\alpha\beta}\Phi_{\gamma\gamma 0} \\
&\quad + 60\gamma_{\alpha\beta}\partial^2\Phi_{\gamma\gamma 0} - 30\gamma_{\alpha\beta}\partial^2\Phi_{000} - 30\gamma_{\alpha\beta}\partial^2_{\gamma\delta}\Phi_{\gamma\delta 0} - 12\partial^2_{\alpha\beta}b_0 \\
&\quad - 24\gamma_{\alpha\beta}\partial^2 b_0 + 6\partial^2_{\alpha\beta}v_0 + 6\gamma_{\alpha\beta}\partial^2 v_0 + 60\partial_{(\alpha}h_{\beta)0} \\
&\quad + 90\gamma_{\alpha\beta}\partial_\gamma h_{\gamma 0} + 15\gamma_{\alpha\beta}\Phi_{000} - 45\gamma_{\alpha\beta}\Phi_{\gamma\gamma 0}\ . \qquad (19)
\end{aligned}
$$

The first and second step constraints besides (18) have the following non-trivial Poisson brackets

$$
\begin{aligned}
\{\overset{(1)}{C}{}^{\lambda}(x), \overset{(2)}{C}{}^{v}(y)\} &= \delta(x-y), \\
\{\overset{(1)}{C}{}^{\varphi}(x)\ \overset{(2)}{C}{}^{\lambda}_{\alpha}(y)\} &= \partial^y_\alpha\delta(x-y), \\
\{\overset{(1)}{C}{}^{v}_{\alpha}(x)\ \overset{(2)}{C}{}^{\lambda}_{\beta}(y)\} &= \delta_{\alpha\beta}(x-y), \\
\{\overset{(1)}{C}{}^{\lambda}_{\alpha}(x)\ \overset{(2)}{C}{}^{v}_{\beta}(y)\} &= -\delta_{\alpha\beta}(x-y)\ . \qquad (20)
\end{aligned}
$$

where $\partial^y_\alpha = \frac{\partial}{\partial y_\alpha}$.

At the third stage new constraints do not emerge but the partial determination of the Lagrange multipliers happens

$$\Lambda^v_\alpha = -\partial_\alpha v_0,\ \Lambda^\lambda_\alpha = 0.$$

Thus, we have 15 "non-commutative" constraints. These are the first step constraints $\overset{(1)}{C}{}^{\varphi}$, $\overset{(1)}{C}{}^{\alpha}$, $\overset{(1)}{C}{}^{\lambda}_{\alpha}$, $\overset{(1)}{C}{}^{v}_{\alpha}$ and the second step ones $\overset{(2)}{C}{}^{\lambda}_{\alpha}$, $\overset{(2)}{C}{}^{\varphi}$ $\overset{(2)}{C}{}^{\varphi}_{\alpha}$. Among these constraints there is a linear combination, that has zero brackets with all other constraints, i.e., it is the first kind constraint

$$\overset{(1)}{C}{}^{c} + \partial_\alpha\overset{(1)}{C}{}^{v}_{\alpha} + \overset{(2)}{C}{}^{v},$$

thus, among 15 "non-commutative" constraints, there are only 14 second kind ones.

Having computed the constraint algebra, let us calculate the number of degrees of freedom. The number of all field components in theory (14) equals $20+10+4+1+4+4 = 43$. In this, there are 22 first and 20 second step constraints. Among them, there are 28 first and 14 second kind constraints. Hence the number of degrees of freedom for this theory equals $43 - 28 - \frac{14}{2} = 8$ and not 7 as for the theory describing the massive particle of spin 3.

Thus, one can conclude that the presence of derivatives in field redefinitions, as in (13) for example, can result in the change of the number of degrees of freedom in the theory.

4 Conclusion

Thus, in this paper we built the canonical Hamiltonians and full systems of constraints for the free massive fields of spin 2 and 3. We have showen that at substitutions of variables with use of derivatives the number of physical degrees of freedom in theory will be able to change. Of course it is not mean that such changes always happen. It implies that the use derivatives in substitutions of variables requires more careful examination.

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References

[1] Yu.M. Zinoviev, *"Gauge Invariant Description of Massive High Spin Particles",* IHEP preprint 83-91, Protvino, 1983.

[2] S.M. Klishevich, Yu.M. Zinoviev, *"On lectromagnetic Interaction of Massive Spin-2 Particle",* IHEP preprint 97-41, Protvino, 1997.

[3] P.A.M. Dirac. Proc. Roy. Soc., **A246**, 326 (1958).

[4] A.J. Hanson, T. Regge, C. Teitelboim. *Constrained Hamiltonian Systems.* Rome, Accademia Nazionale dei Lincei, 1976.

A

The canonical Hamiltonian for the Lagrangian (7) has the following form:

$$
\begin{aligned}
\mathcal{H} \;=\; & \frac{1}{5}\bar{p}^{\Phi}_{000}p^{\Phi}_{000} + \frac{1}{10}\bar{p}^{\Phi}_{\alpha\beta\gamma}p^{\Phi}_{\alpha\beta\gamma} - \frac{3}{50}\bar{p}^{\Phi}_{\beta\gamma\gamma}p^{\Phi}_{\alpha\alpha\beta} + \frac{1}{20}\bar{p}^{\Phi}_{\alpha\alpha\beta}p^{b}_{\beta} + \frac{1}{20}\bar{p}^{b}_{\beta}p^{\Phi}_{\alpha\alpha\beta} \\
& + \frac{1}{30}\bar{p}^{h}_{\alpha\beta}p^{h}_{\alpha\beta} - \frac{7}{720}\bar{p}^{h}_{\beta\beta}p^{h}_{\alpha\alpha} + \frac{1}{12}\bar{p}^{h}_{\alpha\alpha}p^{\varphi} + \frac{1}{12}p^{h}_{\alpha\alpha}\bar{p}^{\varphi} + \frac{1}{6}\bar{p}^{b}_{\alpha}p^{b}_{\alpha} + \bar{p}^{\varphi}p^{\varphi} \\
& - 3\partial_{\beta}\Phi_{\alpha\alpha\beta}\bar{p}^{\Phi}_{000} - 3\partial_{\beta}\bar{\Phi}_{\alpha\alpha\beta}p^{\Phi}_{000} + 3\partial_{\alpha}\Phi_{\alpha00}\bar{p}^{\Phi}_{000} + 3\partial_{\alpha}\bar{\Phi}_{\alpha00}p^{\Phi}_{000} \\
& + \frac{3}{5}\partial_{\alpha}\Phi_{\alpha\beta0}\bar{p}^{\Phi}_{\beta\gamma\gamma} + \frac{3}{5}\partial_{\alpha}\bar{\Phi}_{\alpha\beta0}p^{\Phi}_{\beta\gamma\gamma} + \frac{6}{5}\partial_{\alpha}b_{\alpha}\bar{p}^{\Phi}_{000} + \frac{6}{5}\partial_{\alpha}\bar{b}_{\alpha}p^{\Phi}_{000} \\
& + \frac{9}{25}\partial_{\alpha}b_{0}\bar{p}^{\Phi}_{\alpha\beta\beta} + \frac{9}{25}\partial_{\alpha}\bar{b}_{0}p^{\Phi}_{\alpha\beta\beta} + \frac{7}{24}\partial_{\alpha}h_{\alpha0}\bar{p}^{h}_{\beta\beta} + \frac{7}{24}\partial_{\alpha}\bar{h}_{\alpha0}p^{h}_{\beta\beta} \\
& - \frac{1}{2}\partial_{\alpha}\Phi_{\alpha\beta0}\bar{p}^{b}_{\beta} - \frac{1}{2}\partial_{\alpha}\bar{\Phi}_{\alpha\beta0}p^{b}_{\beta} + \frac{6}{5}\partial_{\alpha}b_{0}\bar{p}^{b}_{\alpha} + \frac{6}{5}\partial_{\alpha}\bar{b}_{0}p^{b}_{\alpha} - \frac{5}{2}\partial_{\alpha}h_{\alpha0}\bar{p}^{\varphi} \\
& - \frac{5}{2}\partial_{\alpha}\bar{h}_{\alpha0}p^{\varphi} + \frac{7}{48}\bar{\Phi}_{000}p^{h}_{\alpha\alpha} + \frac{7}{48}\Phi_{000}\bar{p}^{h}_{\alpha\alpha} + \bar{\Phi}_{\alpha\beta0}p^{h}_{\alpha\beta} + \Phi_{\alpha\beta0}\bar{p}^{h}_{\alpha\beta} \\
& - \frac{7}{16}\bar{\Phi}_{\alpha\alpha0}p^{h}_{\beta\beta} - \frac{7}{16}\Phi_{\alpha\alpha0}\bar{p}^{h}_{\beta\beta} + \frac{3}{40}\bar{b}_{0}p^{h}_{\alpha\alpha} + \frac{3}{40}b_{0}\bar{p}^{h}_{\alpha\alpha} - \frac{5}{4}\bar{\Phi}_{000}p^{\varphi} \\
& - \frac{5}{4}\Phi_{000}\bar{p}^{\varphi} + \frac{15}{4}\bar{\Phi}_{\alpha\alpha0}p^{\varphi} + \frac{15}{4}\Phi_{\alpha\alpha0}\bar{p}^{\varphi} + \frac{9}{2}\bar{b}_{0}p^{\varphi} + \frac{9}{2}b_{0}\bar{p}^{\varphi}
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&+ 30\partial_\beta\Phi_{\alpha\alpha\beta}\partial_\delta\bar{\Phi}_{\gamma\gamma\delta} - 30\partial_\beta\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{\Phi}_{\gamma00} - 12\partial_\beta\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{b}_\gamma \\
&+ 10\partial_\delta\Phi_{\alpha\beta\gamma}\partial_\delta\bar{\Phi}_{\alpha\beta\gamma} + 30\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\delta\bar{\Phi}_{\beta\gamma\delta} - 30\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{\Phi}_{\beta\delta\delta} \\
&+ 30\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{\Phi}_{\beta00} + 12\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{b}_\beta - 30\partial_\delta\bar{\Phi}_{\beta\gamma\delta}\partial_\alpha\Phi_{\alpha\beta\gamma} \\
&+ 30\partial_\gamma\bar{\Phi}_{\beta\delta\delta}\partial_\alpha\Phi_{\alpha\beta\gamma} - 30\partial_\gamma\bar{\Phi}_{\beta00}\partial_\alpha\Phi_{\alpha\beta\gamma} - 12\partial_\alpha\Phi_{\alpha\beta\gamma}\partial_\gamma\bar{b}_\beta \\
&- 12\partial_\beta\bar{\Phi}_{\alpha\alpha\beta}\partial_\gamma b_\gamma + 12\partial_\gamma\bar{\Phi}_{\alpha\alpha\beta}\partial_\gamma b_\beta - 12\partial_\alpha\bar{\Phi}_{\alpha\beta\gamma}\partial_\gamma b_\beta \\
&- 30\partial_\beta\Phi_{\alpha\alpha0}\partial_\gamma\bar{\Phi}_{\beta\gamma0} + 30\partial_\beta\Phi_{\alpha\alpha0}\partial_\beta\bar{\Phi}_{\gamma\gamma0} - 30\partial_\beta\Phi_{\alpha\alpha0}\partial_\beta\bar{\Phi}_{000} \\
&- 12\partial_\beta\Phi_{\alpha\alpha0}\partial_\beta\bar{b}_0 + 24\partial_\gamma\bar{\Phi}_{\beta\gamma0}\partial_\alpha\Phi_{\alpha\beta0} - 30\partial_\beta\bar{\Phi}_{\gamma\gamma0}\partial_\alpha\Phi_{\alpha\beta0} \\
&+ 30\partial_\beta\bar{\Phi}_{000}\partial_\alpha\Phi_{\alpha\beta0} + \frac{42}{5}\partial_\alpha\Phi_{\alpha\beta0}\partial_\beta\bar{b}_0 - 30\partial_\gamma\Phi_{\alpha\beta0}\partial_\gamma\bar{\Phi}_{\alpha\beta0} \\
&- 12\partial_\beta\bar{\Phi}_{\alpha\alpha0}\partial_\beta b_0 + \frac{42}{5}\partial_\alpha\bar{\Phi}_{\alpha\beta0}\partial_\beta b_0 - 12\partial_\beta\bar{\Phi}_{\alpha00}\partial_\beta b_\alpha \\
&- 30\partial_\beta\Phi_{\alpha00}\partial_\gamma\bar{\Phi}_{\alpha\beta\gamma} + 30\partial_\beta\Phi_{\alpha00}\partial_\beta\bar{\Phi}_{\alpha\gamma\gamma} - 12\partial_\beta\Phi_{\alpha00}\partial_\beta\bar{b}_\alpha \\
&- 30\partial_\alpha\Phi_{\alpha00}\partial_\gamma\bar{\Phi}_{\beta\beta\gamma} + 12\partial_\alpha\Phi_{\alpha00}\partial_\beta\bar{b}_\beta + 12\partial_\alpha\bar{\Phi}_{\alpha00}\partial_\beta b_\beta \\
&+ 20\partial_\alpha\bar{\Phi}_{000}\partial_\alpha\Phi_{000} + 12\partial_\alpha\bar{\Phi}_{000}\partial_\alpha b_0 + 30\partial_\alpha\Phi_{000}\partial_\beta\bar{\Phi}_{\alpha\beta0} \\
&- 30\partial_\alpha\Phi_{000}\partial_\alpha\bar{\Phi}_{\beta\beta0} + 12\partial_\alpha\Phi_{000}\partial_\alpha\bar{b}_0 - \frac{36}{5}\partial_\beta b_\alpha\partial_\alpha\bar{b}_\beta \\
&+ \frac{36}{5}\partial_\alpha b_\alpha\partial_\beta\bar{b}_\beta + \frac{216}{25}\partial_\alpha b_0\partial_\alpha\bar{b}_0 + 30\partial_\gamma h_{\alpha\beta}\partial_\gamma\bar{h}_{\alpha\beta} - 30\partial_\gamma h_{\alpha\beta}\bar{\Phi}_{\alpha\beta\gamma} \\
&- 30\partial_\gamma\bar{h}_{\alpha\beta}\Phi_{\alpha\beta\gamma} + 30\partial_\beta h_{\alpha\alpha}\partial_\gamma\bar{h}_{\beta\gamma} - 30\partial_\beta h_{\alpha\alpha}\partial_\beta\bar{h}_{\gamma\gamma} + 5\partial_\beta h_{\alpha\alpha}\partial_\beta\bar{\varphi} \\
&+ 30\partial_\beta h_{\alpha\alpha}\partial_\beta\bar{h}_{00} + 15\partial_\beta h_{\alpha\alpha}\bar{\Phi}_{\beta00} - 15\partial_\beta h_{\alpha\alpha}\bar{\Phi}_{\beta\gamma\gamma} - 18\partial_\beta h_{\alpha\alpha}\bar{b}_\beta \\
&- 60\partial_\gamma\bar{h}_{\beta\gamma}\partial_\alpha h_{\alpha\beta} + 30\partial_\beta\bar{h}_{\gamma\gamma}\partial_\alpha h_{\alpha\beta} - 5\partial_\beta\bar{\varphi}\partial_\alpha h_{\alpha\beta} - 30\partial_\beta\bar{h}_{00}\partial_\alpha h_{\alpha\beta} \\
&- 60\partial_\alpha h_{\alpha\beta}\bar{\Phi}_{\beta00} + 60\partial_\alpha h_{\alpha\beta}\bar{\Phi}_{\beta\gamma\gamma} + 5\partial_\beta\bar{h}_{\alpha\alpha}\partial_\beta\varphi + 15\partial_\beta\bar{h}_{\alpha\alpha}\Phi_{\beta00} \\
&- 15\partial_\beta\bar{h}_{\alpha\alpha}\Phi_{\beta\gamma\gamma} - 18\partial_\beta\bar{h}_{\alpha\alpha}b_\beta - 5\partial_\beta\varphi\partial_\alpha\bar{h}_{\alpha\beta} - 60\partial_\alpha\bar{h}_{\alpha\beta}\Phi_{\beta00} \\
&+ 60\partial_\alpha\bar{h}_{\alpha\beta}\Phi_{\beta\gamma\gamma} + \frac{85}{4}\partial_\alpha h_{\alpha0}\partial_\beta\bar{h}_{\beta0} + \frac{325}{8}\partial_\alpha h_{\alpha0}\bar{\Phi}_{000} - \frac{255}{8}\partial_\alpha h_{\alpha0}\bar{\Phi}_{\beta\beta0} \\
&+ \frac{63}{4}\partial_\alpha h_{\alpha0}\bar{b}_0 - 60\partial_\beta h_{\alpha0}\partial_\beta\bar{h}_{\alpha0} + 60\partial_\beta h_{\alpha0}\bar{\Phi}_{\alpha\beta0} + 60\partial_\beta\bar{h}_{\alpha0}\Phi_{\alpha\beta0} \\
&+ \frac{325}{8}\partial_\alpha\bar{h}_{\alpha0}\Phi_{000} - \frac{255}{8}\partial_\alpha\bar{h}_{\alpha0}\Phi_{\beta\beta0} + \frac{63}{4}\partial_\alpha\bar{h}_{\alpha0}b_0 - \frac{1}{4}\partial_\alpha\varphi\partial_\alpha\bar{\varphi} \\
&- 5\partial_\alpha\varphi\partial_\alpha\bar{h}_{00} - \frac{5}{2}\partial_\alpha\varphi\bar{\Phi}_{\alpha00} + \frac{5}{2}\partial_\alpha\varphi\bar{\Phi}_{\alpha\beta\beta} - 5\partial_\alpha\bar{\varphi}\partial_\alpha h_{00} - \frac{5}{2}\partial_\alpha\bar{\varphi}\Phi_{\alpha00}
\end{aligned}
$$

$$+\frac{5}{2}\partial_\alpha\bar{\varphi}\Phi_{\alpha\beta\beta} - 45\partial_\alpha\bar{h}_{00}\Phi_{\alpha 00} + 15\partial_\alpha\bar{h}_{00}\Phi_{\alpha\beta\beta} + 18\partial_\alpha\bar{h}_{00}b_\alpha$$
$$- 30\partial_\alpha h_{00}\partial_\beta\bar{h}_{\alpha\beta} + 30\partial_\alpha h_{00}\partial_\alpha\bar{h}_{\beta\beta} - 45\partial_\alpha h_{00}\bar{\Phi}_{\alpha 00} + 15\partial_\alpha h_{00}\bar{\Phi}_{\alpha\beta\beta}$$
$$+ 18\partial_\alpha h_{00}\bar{b}_\alpha + 10\bar{\Phi}_{\alpha\beta\gamma}\Phi_{\alpha\beta\gamma} + 30\bar{\Phi}_{\beta 00}\Phi_{\alpha\alpha\beta} - \frac{255}{16}\bar{\Phi}_{000}\Phi_{\alpha\alpha 0}$$
$$+ \frac{165}{16}\bar{\Phi}_{000}\Phi_{000} + \frac{63}{8}\bar{\Phi}_{000}b_0 - \frac{255}{16}\bar{\Phi}_{\alpha\alpha 0}\Phi_{000} - \frac{45}{8}\bar{\Phi}_{\alpha\alpha 0}b_0 - \frac{45}{8}\Phi_{\alpha\alpha 0}\bar{b}_0$$
$$- 30\bar{\Phi}_{\beta\gamma\gamma}\Phi_{\alpha\alpha\beta} + \frac{45}{16}\bar{\Phi}_{\beta\beta 0}\Phi_{\alpha\alpha 0} + 30\bar{\Phi}_{\alpha\beta\beta}\Phi_{\alpha 00} + \frac{63}{8}\Phi_{000}\bar{b}_0$$
$$+ 45\bar{h}_{00}h_{\alpha\alpha} - 45\bar{h}_{00}h_{00} + 45\bar{h}_{\alpha\alpha}h_{00} - 45\bar{h}_{\beta\beta}h_{\alpha\alpha} + \frac{81}{20}\bar{b}_0 b_0.$$

B

Canonical Hamiltonian, corresponding to Lagrangian (14), has the form

$$\begin{aligned}
\mathcal{H} \;=\; & \frac{1}{10}\bar{p}^\Phi_{\alpha\beta\gamma}p^\Phi_{\alpha\beta\gamma} - \frac{3}{50}\bar{p}^\Phi_{\beta\gamma\gamma}p^\Phi_{\alpha\alpha\beta} + \frac{1}{6}\bar{p}^\Phi_{000}p^b_0 + \frac{1}{6}\bar{p}^b_0 p^\Phi_{000} + \frac{1}{20}\bar{p}^\Phi_{\alpha\alpha\beta}p^b_\beta \\
& + \frac{1}{20}\bar{p}^b_\beta p^\Phi_{\alpha\alpha\beta} + \frac{1}{30}\bar{p}^h_{\alpha\beta}p^h_{\alpha\beta} - \frac{1}{60}\bar{p}^h_{\beta\beta}p^h_{\alpha\alpha} - \frac{5}{36}\bar{p}^b_0 p^b_0 + \frac{1}{6}\bar{p}^b_\alpha p^b_\alpha \\
& - \partial_\beta\Phi_{\alpha\alpha\beta}\bar{p}^\Phi_{000} - \partial_\beta\bar{\Phi}_{\alpha\alpha\beta}p^\Phi_{000} - \partial_\alpha\Phi_{\alpha 00}\bar{p}^\Phi_{000} - \partial_\alpha\bar{\Phi}_{\alpha 00}p^\Phi_{000} \qquad (22) \\
& + \frac{3}{5}\partial_\alpha\Phi_{\alpha\beta 0}\bar{p}^\Phi_{\beta\gamma\gamma} + \frac{3}{5}\partial_\alpha\bar{\Phi}_{\alpha\beta 0}p^\Phi_{\beta\gamma\gamma} + \frac{1}{2}\partial_\alpha h_{\alpha 0}\bar{p}^h_{\beta\beta} + \frac{1}{2}\partial_\alpha\bar{h}_{\alpha 0}p^h_{\beta\beta} \\
& + \frac{1}{4}\bar{\Phi}_{000}p^h_{\alpha\alpha} + \frac{1}{4}\Phi_{000}\bar{p}^h_{\alpha\alpha} + \bar{\Phi}_{\alpha\beta 0}p^h_{\alpha\beta} + \Phi_{\alpha\beta 0}\bar{p}^h_{\alpha\beta} - \frac{3}{4}\bar{\Phi}_{\alpha\alpha 0}p^h_{\beta\beta} \\
& - \frac{3}{4}\Phi_{\alpha\alpha 0}\bar{p}^h_{\beta\beta} - \frac{5}{3}\partial_\beta\Phi_{\alpha\alpha\beta}\bar{p}^b_0 - \frac{5}{3}\partial_\beta\bar{\Phi}_{\alpha\alpha\beta}p^b_0 - \frac{1}{2}\partial_\alpha\Phi_{\alpha\beta 0}\bar{p}^b_\beta - \frac{1}{2}\partial_\alpha\bar{\Phi}_{\alpha\beta 0}p^b_\beta \\
& + \frac{10}{3}\partial_\alpha\Phi_{\alpha 00}\bar{p}^b_0 + \frac{10}{3}\partial_\alpha\bar{\Phi}_{\alpha 00}p^b_0 + \partial_\alpha b_\alpha\bar{p}^b_0 + \partial_\alpha\bar{b}_\alpha p^b_0 - \frac{1}{2}\partial_\alpha v_\alpha\bar{p}^b_0 \\
& - \frac{1}{2}\partial_\alpha\bar{v}_\alpha p^b_0 - \frac{1}{6}\partial_\alpha\bar{v}_0 p^b_\alpha - \frac{1}{6}\partial_\alpha v_0\bar{p}^b_\alpha - 2\partial_\alpha\bar{\Phi}_{000}\partial_\alpha v_0 - 2\partial_\alpha\Phi_{000}\partial_\alpha\bar{v}_0 \\
& + 2\partial_\beta\bar{\Phi}_{\alpha 00}\partial_\beta v_\alpha + 2\partial_\beta\Phi_{\alpha 00}\partial_\beta\bar{v}_\alpha - 2\partial_\beta\bar{\Phi}_{\alpha 00}\partial_\alpha v_\beta - 2\partial_\beta\Phi_{\alpha 00}\partial_\alpha\bar{v}_\beta \\
& + 6\partial_\alpha\Phi_{\alpha 00}\partial_\beta\bar{v}_\beta + 6\partial_\alpha\bar{\Phi}_{\alpha 00}\partial_\beta v_\beta + 2\partial_\beta\Phi_{\alpha\alpha 0}\partial_\beta\bar{v}_0 + 2\partial_\beta\bar{\Phi}_{\alpha\alpha 0}\partial_\beta v_0 \\
& - 4\partial_\alpha\Phi_{\alpha\beta 0}\partial_\beta\bar{v}_0 - 4\partial_\alpha\bar{\Phi}_{\alpha\beta 0}\partial_\beta v_0 + 2\partial_\alpha\Phi_{\alpha\beta\gamma}\partial_\gamma\bar{v}_\beta + 2\partial_\alpha\bar{\Phi}_{\alpha\beta\gamma}\partial_\gamma v_\beta \\
& - 2\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\gamma\bar{v}_\beta - 2\partial_\gamma\bar{\Phi}_{\alpha\alpha\beta}\partial_\gamma v_\beta + 2\partial_\gamma\Phi_{\alpha\alpha\beta}\partial_\beta\bar{v}_\gamma + 2\partial_\gamma\bar{\Phi}_{\alpha\alpha\beta}\partial_\beta v_\gamma \\
& + 20\partial_\alpha\bar{\Phi}_{000}\partial_\alpha\Phi_{000} - 30\partial_\beta\Phi_{\alpha\alpha 0}\partial_\beta\bar{\Phi}_{000} - 30\partial_\alpha\Phi_{000}\partial_\alpha\bar{\Phi}_{\beta\beta 0} \\
& + 30\partial_\alpha\Phi_{000}\partial_\beta\bar{\Phi}_{\alpha\beta 0} + 30\partial_\beta\bar{\Phi}_{000}\partial_\alpha\Phi_{\alpha\beta 0} - 80\partial_\beta\bar{\Phi}_{\beta 00}\partial_\alpha\Phi_{\alpha 00} \\
& - 30\partial_\beta\Phi_{\alpha\alpha 0}\partial_\gamma\bar{\Phi}_{\beta\gamma 0} - 30\partial_\beta\bar{\Phi}_{\gamma\gamma 0}\partial_\alpha\Phi_{\alpha\beta 0} + 30\partial_\beta\Phi_{\alpha\alpha 0}\partial_\beta\bar{\Phi}_{\gamma\gamma 0}
\end{aligned}$$

$$
\begin{aligned}
&+ 24\partial_\gamma \bar{\Phi}_{\beta\gamma 0}\partial_\alpha \Phi_{\alpha\beta 0} - 30\partial_\gamma \Phi_{\alpha\beta 0}\partial_\gamma \bar{\Phi}_{\alpha\beta 0} + 10\partial_\beta \Phi_{\alpha\alpha\beta}\partial_\gamma \bar{\Phi}_{\gamma 00} \\
&+ 10\partial_\alpha \Phi_{\alpha 00}\partial_\gamma \bar{\Phi}_{\beta\beta\gamma} - 30\partial_\alpha \Phi_{\alpha\beta\gamma}\partial_\gamma \bar{\Phi}_{\beta 00} - 30\partial_\beta \Phi_{\alpha 00}\partial_\gamma \bar{\Phi}_{\alpha\beta\gamma} \\
&+ 30\partial_\gamma \bar{\Phi}_{\beta 00}\partial_\gamma \Phi_{\alpha\alpha\beta} + 30\partial_\beta \Phi_{\alpha 00}\partial_\beta \bar{\Phi}_{\alpha\gamma\gamma} + 10\partial_\delta \Phi_{\alpha\beta\gamma}\partial_\delta \bar{\Phi}_{\alpha\beta\gamma} \\
&+ 10\partial_\beta \Phi_{\alpha\alpha\beta}\partial_\delta \bar{\Phi}_{\gamma\gamma\delta} - 30\partial_\alpha \Phi_{\alpha\beta\gamma}\partial_\delta \bar{\Phi}_{\beta\gamma\delta} + 30\partial_\alpha \Phi_{\alpha\beta\gamma}\partial_\gamma \bar{\Phi}_{\beta\delta\delta} \\
&+ 30\partial_\delta \bar{\Phi}_{\beta\gamma\delta}\partial_\gamma \Phi_{\alpha\alpha\beta} - 30\partial_\gamma \bar{\Phi}_{\beta\delta\delta}\partial_\gamma \Phi_{\alpha\alpha\beta} + 12\partial_\alpha \bar{\Phi}_{000}\partial_\alpha b_0 \\
&+ 12\partial_\alpha \Phi_{000}\partial_\alpha \bar{b}_0 - 12\partial_\beta \bar{\Phi}_{\alpha 00}\partial_\beta b_\alpha - 12\partial_\beta \Phi_{\alpha 00}\partial_\beta \bar{b}_\alpha \\
&- 12\partial_\alpha \Phi_{\alpha 00}\partial_\beta \bar{b}_\beta - 12\partial_\alpha \bar{\Phi}_{\alpha 00}\partial_\beta b_\beta - 12\partial_\beta \Phi_{\alpha\alpha 0}\partial_\beta \bar{b}_0 \\
&- 12\partial_\beta \bar{\Phi}_{\alpha\alpha 0}\partial_\beta b_0 + 12\partial_\alpha \Phi_{\alpha\beta 0}\partial_\beta \bar{b}_0 + 12\partial_\alpha \bar{\Phi}_{\alpha\beta 0}\partial_\beta b_0 \\
&- 12\partial_\alpha \Phi_{\alpha\beta\gamma}\partial_\gamma \bar{b}_\beta - 12\partial_\alpha \bar{\Phi}_{\alpha\beta\gamma}\partial_\gamma b_\beta + 12\partial_\gamma \Phi_{\alpha\alpha\beta}\partial_\gamma \bar{b}_\beta \\
&+ 12\partial_\gamma \bar{\Phi}_{\alpha\alpha\beta}\partial_\gamma b_\beta - 30\partial_\alpha h_{\alpha\beta}\partial_\beta \bar{h}_{00} - 30\partial_\alpha h_{00}\partial_\beta \bar{h}_{\alpha\beta} \\
&+ 30\partial_\beta \bar{h}_{00}\partial_\beta h_{\alpha\alpha} + 30\partial_\alpha h_{00}\partial_\alpha \bar{h}_{\beta\beta} - 60\partial_\beta h_{\alpha 0}\partial_\beta \bar{h}_{\alpha 0} \\
&+ 15\partial_\alpha h_{\alpha 0}\partial_\beta \bar{h}_{\beta 0} + 30\partial_\gamma h_{\alpha\beta}\partial_\gamma \bar{h}_{\alpha\beta} - 60\partial_\alpha h_{\alpha\beta}\partial_\gamma \bar{h}_{\beta\gamma} \\
&+ 30\partial_\alpha h_{\alpha\beta}\partial_\beta \bar{h}_{\gamma\gamma} + 30\partial_\gamma \bar{h}_{\beta\gamma}\partial_\beta h_{\alpha\alpha} - 30\partial_\beta \bar{h}_{\gamma\gamma}\partial_\beta h_{\alpha\alpha} \\
&+ \frac{75}{2}\partial_\alpha h_{\alpha 0}\bar{\Phi}_{000} + \frac{75}{2}\partial_\alpha \bar{h}_{\alpha 0}\Phi_{000} - 45\partial_\alpha h_{00}\bar{\Phi}_{\alpha 00} - 45\partial_\alpha \bar{h}_{00}\Phi_{\alpha 00} \\
&+ 15\partial_\alpha h_{00}\bar{\Phi}_{\alpha\beta\beta} + 15\partial_\alpha \bar{h}_{00}\Phi_{\alpha\beta\beta} + 60\partial_\beta h_{\alpha 0}\bar{\Phi}_{\alpha\beta 0} + 60\partial_\beta \bar{h}_{\alpha 0}\Phi_{\alpha\beta 0} \\
&- \frac{45}{2}\partial_\alpha h_{\alpha 0}\bar{\Phi}_{\beta\beta 0} - \frac{45}{2}\partial_\alpha \bar{h}_{\alpha 0}\Phi_{\beta\beta 0} - 60\partial_\alpha h_{\alpha\beta}\bar{\Phi}_{\beta 00} - 60\partial_\alpha \bar{h}_{\alpha\beta}\Phi_{\beta 00} \\
&+ 15\partial_\beta h_{\alpha\alpha}\bar{\Phi}_{\beta 00} + 15\partial_\beta \bar{h}_{\alpha\alpha}\Phi_{\beta 00} - 30\partial_\gamma h_{\alpha\beta}\bar{\Phi}_{\alpha\beta\gamma} - 30\partial_\gamma \bar{h}_{\alpha\beta}\Phi_{\alpha\beta\gamma} \\
&+ 60\partial_\alpha h_{\alpha\beta}\bar{\Phi}_{\beta\gamma\gamma} + 60\partial_\alpha \bar{h}_{\alpha\beta}\Phi_{\beta\gamma\gamma} - 15\partial_\beta h_{\alpha\alpha}\bar{\Phi}_{\beta\gamma\gamma} - 15\partial_\beta \bar{h}_{\alpha\alpha}\Phi_{\beta\gamma\gamma} \\
&+ \frac{35}{4}\bar{\Phi}_{000}\Phi_{000} - \frac{45}{4}\bar{\Phi}_{000}\Phi_{\alpha\alpha 0} - \frac{45}{4}\bar{\Phi}_{\alpha\alpha 0}\Phi_{000} + 30\bar{\Phi}_{\beta 00}\Phi_{\alpha\alpha\beta} \\
&+ 30\bar{\Phi}_{\alpha\beta\beta}\Phi_{\alpha 00} - \frac{45}{4}\bar{\Phi}_{\beta\beta 0}\Phi_{\alpha\alpha 0} + 10\bar{\Phi}_{\alpha\beta\gamma}\Phi_{\alpha\beta\gamma} - 30\bar{\Phi}_{\beta\gamma\gamma}\Phi_{\alpha\alpha\beta} \\
&+ \bar{v}_0\lambda_0 + v_0\bar{\lambda}_0 - \bar{v}_\alpha\lambda_\alpha - v_\alpha\bar{\lambda}_\alpha + \partial_\alpha\varphi\bar{\lambda}_\alpha + \partial_\alpha\bar{\varphi}\lambda_\alpha \ .
\end{aligned}
$$